Manual of nabla_checker

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1 How to use nabla_checker

For a given function $F : \mathbb{R}^n \to \mathbb{R}^m$, many optimization algorithm requires the closed form of the transposed Jacobian $\nabla F : \mathbb{R}^n \to \mathbb{R}^{n\times m}$. When you calculate $\nabla F(x) \in \mathbb{R}^{n\times m}$ in an algebraic manner, `nabla_checker.m` judges whether the calculation is correct or not.

Usage: `[] = nabla_checker(FUNC,nabFUNC,n,m)`

- `FUNC` — implies the given function $F : \mathbb{R}^n \to \mathbb{R}^m$.
- `nabFUNC` — implies the function which is expected to be $\nabla F : \mathbb{R}^n \to \mathbb{R}^{n\times m}$.
- `n` — implies the value of n, i.e., the dimension of the argument x of function F.
- `m` — implies the value of m, i.e., the dimension of $F(x)$.

2 What nabla_checker is doing

`nabla_checker.m` actually evaluates the value of

$$\begin{aligned} AE(x) &= \|D_F(x) - J(x)\|, \\ RE(x) &= \frac{\|D_F(x) - J(x)\|}{\|D_F(x)\|} \end{aligned}$$

for 10 random choices of $x \in \mathbb{R}^n$, where $J : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is the closed form function expected to be ∇F, and $D_F(x)$ is the finite difference approximation of $\nabla F(x)$, i.e., $n \times m$-dimensional matrix whose component is

$$D_F(x)_{ij} = \frac{F_j(x + \tau e^i) - F_j(x)}{\tau} \qquad (\tau = 10^{-6}).$$

When the mean value of $RE(x)$ is less than τ, then `nabla_checker.m` judges that $J(x)$ is equal to $\nabla F(x)$.

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